EVENTS AND PROBABILITIES

Definitions

- A process is random if it is known that when it takes place, one outcome from a possible set of outcomes is sure to occur, but it is impossible to predict with certainty which outcome that will be.
- A sample space is the set of all possible outcomes of a random process or experiment.
- An event is a subset of a sample space.

Equally Likely Probability Formula

- If S is a finite sample space in which all outcomes are equally likely, and E is an event in S, then the probability of E, denoted P(E) is

$$P(E) = \frac{\text{the number of outcomes in E}}{\text{the total number of outcomes in S}}$$

Probability Properties

Let S be a sample space and A, B events in S

- $0 \leq P(A), P(B) \leq 1$
- $P(\varnothing) = 0$ and $P(S) = 1$
- $P(A^c) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Corollary: if A and B are disjoint, then $P(A \cup B) = P(A) + P(B)$

Expected Value

- If the possible outcomes of an experiment, or random process, are real numbers a_1 to a_n, which occur with probabilities p_1 to p_n respectively, then the expected value of the process is $\sum_{k=1}^{n} a_k p_k$

CONDITIONAL PROBABILITY

Definition

- Let A and B be events in a sample space S. If $P(A) \neq 0$, then the conditional probability of B given A, denoted $P(B|A)$ is $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Properties:

- $P(B^c|A) = 1 - P(B|A)$
- Baye's Theorem: Given two events A and B s.t. $P(B) \neq 0$, $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- General Baye's Theorem: Suppose that a sample space S is a union of mutually disjoint events $A_1, A_2, \ldots, A_n$, and suppose B is an event in S with $P(B) \neq 0$

  If k is an integer with $1 \leq k \leq n$, then $P(A_k|B) = \frac{P(B|A_k)P(A_k)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$

INDEPENDENT EVENTS

Let A, B and C be events in a sample space S.

- A and B are independent iff $P(A \cap B) = P(A).P(B)$
- If A and B are independent events, then so are A and B^c
- If $P(A) \neq 0$ and $P(B)\ ) \neq 0$ but $A \cap B = \emptyset$, then the events A and B are not independent
- A, B, and C are pairwise independent iff they satisfy all the following conditions:
  - $P(A \cap B) = P(A).P(B)$
  - $P(B \cap C) = P(B).P(C)$
  - $P(C \cap A) = P(C).P(A)$
- A, B, and C are mutually independent iff they are pairwise independent and $P(A \cap B \cap C) = P(A).P(B).P(C)$
- Generally: events A_1 to A_n are mutually independent iff the probability of the intersection of any subset of the events is the product of the probabilities of the events in the subset.